$\boxed{\text{Day 1 - PM}}$

Prove that $\lim_{x\to 3} \frac{x^2-9}{x-3} = 6$

SCRATCH

let $\varepsilon > 0$ assume $|x-3| < \delta$

want $\left|\frac{x^2-9}{x-3} - 6\right| < \varepsilon$

$|x+3-6| < \varepsilon$

$|x-3| < \varepsilon = \delta$ choose $\delta = \varepsilon$

Pf: let $\varepsilon > 0$. Choose $\delta = \varepsilon$. Assume $|x-3| < \delta$.
Then $\left|\frac{x^2-9}{x-3} - 6\right| = |x+3-6| = |x-3| < \delta = \varepsilon$ ∎

Sided Limits

$\lim_{x\to c^-} f(x) = L$ $\forall \varepsilon > 0 \ \exists \delta > 0$ s.t. $c - x < \delta \Rightarrow |f(x) - L| < \varepsilon$

"left sided"

$\lim_{x\to c^+} f(x) = L$ $\forall \varepsilon > 0 \ \exists \delta > 0$ s.t. $x - c < \delta \Rightarrow |f(x) - L| < \varepsilon$

"right sided

Ex: $\left.\begin{array}{l} \lim_{x\to 0^-} \frac{|x|}{x} = -1 \\ \lim_{x\to 0^+} \frac{|x|}{x} = 1 \end{array}\right\}$ $\frac{|x|}{x}$ is not continuous at 0!

Infinite Limits

$\lim_{x\to\infty} f(x) = L$ "end behavior"

for every $\varepsilon > 0$, there exists $M > 0$ so that $x > M$ implies $|f(x) - L| < \varepsilon$

$\longrightarrow$

$\lim_{x \to \infty} f(x) = \infty$

for every $N > 0$, there exists $M > 0$ such that $x > M$ implies $f(x) > N$.

* Properties of Limits – see handout on website

Squeeze Theorem

if $f(x) \leq g(x) \leq h(x)$ on $|x-c| < \delta$ for some δ and

$\lim_{x \to c} f(x) = L = \lim_{x \to c} h(x)$ then

$$\lim_{x \to c} g(x) = L$$

Ex: $-|x| < x^2 < |x|$ on $(-1,1)$

Since $\lim_{x \to 0} -|x| = 0 = \lim_{x \to 0} |x|$

then $\lim_{x \to 0} x^2 = 0$ by Squeeze Thm.